ON CENTRAL COMMUTATOR GALOIS EXTENSIONS OF RINGS

GEORGE SZETO and LIANYONG XUE

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Abstract. Let $B$ be a ring with 1, $G$ a finite automorphism group of $B$ of order $n$ for some integer $n$, $B^G$ the set of elements in $B$ fixed under each element in $G$, and $\Delta = V_B(B^G)$ the commutator subring of $B^G$ in $B$. Then the type of central commutator Galois extensions is studied. This type includes the types of Azumaya Galois extensions and Galois $H$-separable extensions. Several characterizations of a central commutator Galois extension are given. Moreover, it is shown that when $G$ is inner, $B$ is a central commutator Galois extension of $B^G$ if and only if $B$ is an $H$-separable projective group ring $B^G G_f$. This generalizes the structure theorem for central Galois algebras with an inner Galois group proved by DeMeyer.

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1. Introduction. Galois theory for commutative rings were studied in the sixties and seventies (see [4, Chapter 3]), and several Galois extensions of noncommutative rings were also investigated (see [2, 5, 6, 8]). Recently, central Galois extensions and the DeMeyer-Kanzaki Galois extensions were generalized to the Azumaya Galois extensions and center Galois extensions, respectively (see [1, 9, 10, 11]). $B$ is called an Azumaya Galois extension of $B^G$ with Galois group $G$ if $B$ is a Galois extension of $B^G$ which is an Azumaya algebra over $C^G$ where $C$ is the center of $B$, and $B$ is called a center Galois extension of $B^G$ if $C$ is a Galois algebra with Galois group $G|_C \cong G$. The purpose of the present paper is to study a type of Galois extensions which is strictly between the types of Azumaya Galois extensions and Galois $H$-separable extensions.

Let $\Delta = V_B(B^G)$, the commutator subring of $B^G$ in $B$. We call $B$ a commutator Galois extension of $B^G$ if $\Delta$ is a Galois extension with Galois group $G|_\Delta \cong G$, and $B$ is a central commutator Galois extension of $B^G$ if $\Delta$ is a central Galois algebra with Galois group $G|_\Delta \cong G$. We shall characterize a central commutator Galois extension in terms of a Galois $H$-separable extension $B$ of $B^G$ as studied by Sugano (see [8]) and the $C$-modules $\{J_g \mid g \in G\}$ where $J_g = \{b \in B \mid ba = g(a)b \text{ for all } a \in B\}$. Moreover, it will be shown that $B$ is a central commutator Galois extension of $B^G$ with an inner Galois group $G$ if and only if $B$ is an $H$-separable projective group ring $B^G G_f$ where $B^G G_f = \sum_{g \in G} B^G U_g$ such that $\{U_g \mid g \in G\}$ are free over $B^G$, $bU_g = U_gb$ for all $b \in B^G$ and $g \in G$, and $U_g U_h = U_{gh} f(g,h)$ where $f : G \times G \to \text{units of } C^G$ is a factor set. This generalizes the structure theorem for a central Galois algebra with an inner Galois group proved by DeMeyer (see [3]).
2. Basic definitions and notation. Throughout this paper, $B$ will represent a ring with $1$, $C$ the center of $B$, $G$ a finite automorphism group of $B$ of order $n$ for some integer $n$, $B^G$ the set of elements in $B$ fixed under each element in $G$, and $\Delta = V_B(B^G)$, the commutator subring of $B^G$ in $B$.

Let $A$ be a subring of a ring $B$ with the same identity 1. We call $B$ a separable extension of $A$ if there exist $\{a_i, b_i\}$ in $B$, $i = 1, 2, \ldots, m$ for some integer $m$ such that $\sum a_i b_i = 1$, and $\sum b_i a_i \otimes b_i = \sum a_i \otimes b_i b_i$ for all $b_i$ in $B$ where $\otimes$ is over $A$, and a ring $B$ is called an $H$-separable extension of $A$ if $B \otimes_A B$ is isomorphic to a direct summand of a finite direct sum of $B$ as a $B$-bimodule. An Azumaya algebra is a separable extension of its center. $B$ is called a Galois extension of $B^G$ with Galois group $G$ if there exist elements $\{c_i, d_i\}$ in $B$, $i = 1, 2, \ldots, m$ for some integer $m$ such that $\sum_{i=1}^m c_i g(d_i) = \delta_{1, g}$ for $g \in G$. The set $\{c_i, d_i\}$ is called a $G$-Galois system for $B$. $B$ is called a DeMeyer-Kanzaki Galois extension of $B^G$ if $B$ is an Azumaya $C$-algebra and $C$ is a Galois algebra with Galois group $G|_C \cong G$. If $C$ is a Galois algebra with Galois group $G|_C \cong G$, we call $B$ a center Galois extension of $B^G$. $B$ is called an Azumaya Galois extension if it is a Galois extension of $B^G$ that is an Azumaya $C^G$-algebra, and $B$ is called a Galois $H$-separable extension if it is a Galois and an $H$-separable extension of $B^G$ (see [8]). We call $B$ a commutator Galois extension of $B^G$ if $\Delta$ is a Galois extension with Galois group $G|_\Delta \cong G$, and $B$ is a central commutator Galois extension of $B^G$ if $\Delta$ is a central Galois algebra with Galois group $G|_\Delta \cong G$. For each $g \in G$, let $J_\theta = \{b \in B \mid b x = g(x)b \text{ for all } x \in B\}$ and $J_\theta^A = \{a \in A \mid ax = g(x)a \text{ for all } x \in A\}$ for a subring $A$ of $B$.

3. Central commutator Galois extensions. In this section, we shall give several characterizations of a central commutator Galois extension in terms of Galois $H$-separable extensions and Azumaya Galois extensions, respectively, and prove the converse of a theorem for a Galois $H$-separable extension as given in [8]. We begin with some properties of a commutator Galois extension.

**Lemma 3.1.** If $B$ is a commutator Galois extension of $B^G$, then $\Delta$ is a Galois algebra over $C^G$.

**Proof.** Since $\Delta$ is a Galois extension of $\Delta^G$ with Galois group $G|_\Delta \cong G$, $B$ and $B^G \Delta$ are also Galois extensions of $B^G$ with Galois group $G$ and $G|_{B^G \Delta}$. Thus, the center of $\Delta$ is $C$; and so $\Delta^G = B^G \cap \Delta = C^G$.

**Lemma 3.2.** If $B$ is a commutator Galois extension of $B^G$, then $J_\theta = J_\theta^A$ for each $g \in G$.

**Proof.** Since $J_\theta = \{b \in B \mid ba = g(a)b \text{ for all } a \in B\} \subset \{b \in B \mid ba = g(a)b \text{ for all } a \in B^G\} = \Delta$, $J_\theta \subset J_\theta^A$.

Conversely, for any $x \in J_\theta^A$, $x d = g(d)x$ for all $d \in \Delta$. Since $\Delta$ is a Galois extension of $\Delta^G$ with Galois group $G|_\Delta \cong G$, $B = B^G \Delta$ by the proof of Lemma 3.1. So for any $b \in B$, $b = \sum_{i=1}^m b_i d_i$ for some $b_i \in B^G$, $d_i \in \Delta$ and some integer $m$, we have that $xb = x \sum_{i=1}^m b_i d_i = \sum_{i=1}^m b_i xd_i = \sum_{i=1}^m b_i g(d_i)x = g(\sum_{i=1}^m b_i d_i)x = g(b)x$. Thus, $J_\theta^A \subset J_\theta$.

**Theorem 3.3.** The following are equivalent:

(1) $B$ is a central commutator Galois extension of $B^G$. 

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(2) $B$ is a commutator Galois extension of $B^G$ and $J_g J_{g^{-1}} = C$ for each $g \in G$.

(3) $B$ is a Galois $H$-separable extension of $B^G$, $B = B^G \Delta$, and $n^{-1} \in B$.

**Proof.** (1)$\Rightarrow$(2). It is clear.

(2)$\Rightarrow$(1). By Lemma 3.1, $\Delta^G = C^G$, so $\Delta$ is a Galois algebra with Galois group $G|_{\Delta} \cong G$.

By hypothesis, $J_g J_{g^{-1}} = C$ for each $g \in G$ and by Lemma 3.2, $J_g = J_{g^2}$ for each $g \in G$, so $\Delta$ is a central Galois algebra (see [5, Theorem 1]).

(1)$\Rightarrow$(3). Since $\Delta$ is a central Galois $C^G$-algebra, we have $B = B^G \Delta$, $J_g = J_{g^2}$ for each $g \in G$ by Lemma 3.2 and $J_{g^2} J_{g^{-1}} = C$ (see [6, Lemma 2]). Hence $J_g J_{g^{-1}} = C$ for each $g \in G$. But $B$ is a Galois extension of $B^G$ with the same Galois system for $\Delta$, so $B$ is a Galois $H$-separable extension of $B^G$ (see [8, Theorem 2(iii)$\Rightarrow$(i)]). Moreover, $n^{-1} \in B$ (see [6, Corollary 3]), so (3) holds.

(3)$\Rightarrow$(1). Since $B = B^G \Delta$, the group $H = \{ g \in G | g|_{\Delta} $ is an identity} = \{1\}. Thus, $\Delta$ is a central Galois algebra over $\Delta^G$ (see [8, Theorem 6, (3)(ii)$\Rightarrow$(iii)]) where $\Delta^G = C^G$ by Lemma 3.1.

We remark that (1)$\Rightarrow$(3) in Theorem 3.3 is the converse of [8, Theorem 6]; that is, if $\Delta$ is a central Galois algebra with Galois group $G|_{\Delta} \cong G$,

(i) $n^{-1} \in B$,

(ii) $B = B^G \Delta$, 

(iii) $B$ is a Galois $H$-separable extension of $B^G$. $\square$

In the next theorem, we give a characterization of a central commutator Galois extension in terms of Azumaya Galois extensions.

**Theorem 3.4.** The following are equivalent:

(1) $B$ is a central commutator Galois extension of $B^G$ and $B^G$ is a separable $C^G$-algebra.

(2) $B$ is an Azumaya Galois extension with Galois group $G$.

(3) $B$ is a central commutator Galois extension and a separable extension of $\Delta$.

**Proof.** (1)$\Rightarrow$(2). Since $B$ is a central commutator Galois extension, $B$ is a Galois $H$-separable extension of $B^G$ by Theorem 3.3(3). Thus, $V_B(V_B(B^G)) = B^G$ (see [8, Proposition 4(1)]). This implies that $C \subset B^G$; and so $C = C^G$. Moreover, by Theorem 3.3(3) again, $B = B^G \Delta$, so the center of $B^G$ is $C^G$, the center of $B$. Thus, $B^G$ is an Azumaya $C^G$-algebra. By noting that $B$ is a Galois extension of $B^G$, (2) holds.

(2)$\Rightarrow$(1). It is a consequence of [1, Lemma 1].

(1)$\Rightarrow$(3). Since $B$ is a separable extension of $B^G$ (for it is a Galois extension) and $B^G$ is a separable $C^G$-algebra, $B$ is a separable $C^G$-algebra by the transitivity property of separable extensions. Thus, $B$ is a separable extension of $\Delta$ because $C^G \subset \Delta \subset B$.

(3)$\Rightarrow$(1). Since $\Delta$ is a Galois extension of $\Delta^G$ with Galois group $G|_{\Delta} \cong G$, $\Delta$ is a separable extension of $\Delta^G$. By Lemma 3.1, $\Delta^G = C^G = C$ (for $C$ is the center of $\Delta$). By hypothesis, $B$ is a separable extension of $\Delta$. Hence $B$ is a separable extension of $C$, that is, $B$ is an Azumaya $C$-algebra. By Lemma 3.1 again, $B = B^G \Delta$ such that $B^G$ and $\Delta$ are $C$-subalgebras of the Azumaya $C$-algebra $B$. Hence, they are Azumaya $C$-algebras by the commutator theorem for Azumaya algebras (see [4, Theorem 4.3, page 57]). Since $\Delta$ is a Galois extension of $\Delta^G$ with Galois group $G|_{\Delta} \cong G$, $B$ is a Galois extension of $B^G$ which is an Azumaya $C^G$-algebra. This completes the proof. $\square$
4. H-separable projective group rings. In [3], it was shown that \( B \) is a central Galois algebra with an inner Galois group \( G \) if and only if \( B \) is an Azumaya projective group algebra \( C^G G_f \) over \( C^G \) where \( C^G G_f = \sum_{\mathfrak{g} \in G} C^G U_\mathfrak{g} \) such that \( \{ U_\mathfrak{g} \mid \mathfrak{g} \in G \} \) are free over \( C^G \), \( c U_\mathfrak{g} = U_\mathfrak{g} c \) for all \( c \in C^G \) and \( g \in G \), and \( U_\mathfrak{g} U_h = U_{gh} f(g, h) \), \( f : G \times G \rightarrow \text{units of } C^G \) is a factor set (see [3]). We shall generalize this fact to a central commutator Galois extension with an inner Galois group.

**Theorem 4.1.** \( B \) is a central commutator Galois extension of \( B^G \) with an inner Galois group \( G \) if and only if \( B = B^G G_f \) which is an \( H \)-separable extension of \( B^G \) and \( n^{-1} \in B \).

**Proof.** \((\Rightarrow)\) By Theorem 3.3 \((1) \Rightarrow (3)\), \( B = B^G \Delta \) which is a Galois \( H \)-separable extension of \( B^G \) and \( n^{-1} \in B \), so it suffices to show that \( B = B^G G_f \), a projective group ring with coefficient ring \( B^G \). Since \( \Delta \) is a central Galois \( C^G \)-algebra, by [3, Theorem 2], \( \Delta = C^G G_f \), a projective group algebra over \( C^G \) where \( f : G \times G \rightarrow \text{units of } C^G \) is a factor set such that \( f(g, h) = U_{gh} U_{gh^{-1}} \) for all \( g, h \in G \). Noting that \( b U_\mathfrak{g} = U_\mathfrak{g} b \) for all \( b \in B^G \) and \( g \in G \), we claim that \( \{ U_\mathfrak{g} \mid \mathfrak{g} \in G \} \) are independent over \( B^G \). Assume \( \sum_{\mathfrak{g} \in G} b_\mathfrak{g} U_\mathfrak{g} = 0 \) for some \( b_\mathfrak{g} \in B^G \) and \( g \in G \). Since \( \Delta \) is a Galois extension of \( \Delta \) with Galois group \( G \mid \Delta \cong G \), there exists a \( G \)-Galois system \( \{ c_i, d_i \}, i = 1, 2, \ldots, m \) for some integer \( m \) for \( \mathfrak{g} \) such that \( \sum_{i=1}^m c_i g(d_i) = \delta_{1, g} \) for \( g \in G \). Hence

\[
b_1 = \sum_{\mathfrak{g} \in G} \delta_{1, g} b_\mathfrak{g} U_\mathfrak{g} = \sum_{\mathfrak{g} \in G} c_i g(d_i) b_\mathfrak{g} U_\mathfrak{g} = \sum_{\mathfrak{g} \in G} c_i b_\mathfrak{g} U_\mathfrak{g} d_i = 0.
\]

So \( \sum_{\mathfrak{g} \in G} b_\mathfrak{g} U_\mathfrak{g} = 0 \) for some \( b_\mathfrak{g} \in B^G \) and \( g \in G \) implies that \( b_1 = 0 \). Now for any \( h \in G \), since \( \sum_{\mathfrak{g} \in G} b_\mathfrak{g} U_\mathfrak{g} = 0 \), \( 0 = \sum_{\mathfrak{g} \in G} b_\mathfrak{g} U_{gh^{-1}} = \sum_{\mathfrak{g} \in G} b_\mathfrak{g} f(g, h^{-1}) U_{gh^{-1}} \). Thus, \( b_{gh^{-1}}(h, h^{-1}) = 0 \), and so \( b_{gh^{-1}} = 0 \). This proves that \( \{ U_\mathfrak{g} \mid \mathfrak{g} \in G \} \) are independent over \( B^G \).

\((\Leftarrow)\) Since \( B^G G_f (\cong B^G \otimes_{C^G} C^G G_f) \) is an \( H \)-separable extension of \( B^G \) and \( B^G \) is a direct summand of \( B^G G_f \) as a left \( B^G \)-module, \( V_{B^G G_f}(V_{B^G G_f}(B^G)) = B^G \). This implies that the center of \( B^G G_f \) is \( C^G \). Moreover, \( G \) is inner induced by \( \{ U_\mathfrak{g} \mid \mathfrak{g} \in G \} \), so \( J_\mathfrak{g} = C^G U_\mathfrak{g} \) for each \( g \in G \). But then \( C^G G_f = \oplus \sum_{\mathfrak{g} \in G} C^G U_\mathfrak{g} = \oplus \sum_{\mathfrak{g} \in G} J_\mathfrak{g} \) such that \( J_\mathfrak{g} J_{\mathfrak{g}^{-1}} = (C^G U_\mathfrak{g})(C^G U_{\mathfrak{g}^{-1}}) = C^G \) for all \( g \in G \). By hypothesis, \( n^{-1} \in C^G \), so \( C^G G_f \) is a separable algebra over \( C^G \). Thus, \( \Delta(= C^G G_f) \) is a central Galois algebra (see [5, Theorem 1]) with an inner Galois group \( \tilde{G} \) induced by \( \{ U_\mathfrak{g} \mid \mathfrak{g} \in G \} \). Thus, \( B \) is a central commutator Galois extension of \( B^G \) with an inner Galois group \( G \).

By [7, Theorem 1.2], we derive a one-to-one correspondence between some sets of separable subextensions in a central commutator Galois extension \( B \) of \( B^G \). Let \( \mathcal{F} = \{ \mathcal{A} \mid \mathcal{A} \) is a separable subextension of \( B \) containing \( B^G \) which is a direct summand of \( B \) as a bimodule \} and \( \mathcal{F} = \{ \mathcal{B} \mid \mathcal{B} \) is a separable subalgebra of \( \Delta \) over \( C^G \} \).

**Theorem 4.2.** Let \( B \) be a central commutator Galois extension of \( B^G \). Then, there exists a one-to-one correspondence between \( \mathcal{F} \) and \( \mathcal{F} \) by \( A \rightarrow V_B(A) \).  \( \square \)
**Proof.** By Theorem 3.3(3), $B$ is an $H$-separable extension of $B^G$, so the correspondence holds by [7, Theorem 1.2].

We conclude this paper with two examples of Galois extension $B$ to show that

1. $B$ is a central commutator Galois extension but not an Azumaya Galois extension (see Theorem 3.4),
2. $B$ is a Galois $H$-separable extension but not a central commutator Galois extension (see Theorem 3.3).

**Example 4.3.** Let $A = Q[i,j,k]$ be the quaternion algebra over the rational field $Q$, $B = \left\{ \begin{pmatrix} a_1 & a_2 \\ 0 & a_3 \end{pmatrix} \middle| a_1, a_2, a_3 \in A \right\}$, the ring of all 2-by-2 upper triangular matrices over $A$ and $G = \{1, g_i, g_j, g_k\}$ where $g_i(a) = ia i^{-1}$, $g_j(a) = ja j^{-1}$, $g_k(a) = ka k^{-1}$ for all $a$ in $A$ and $g \begin{pmatrix} a_1 & a_2 \\ 0 & a_3 \end{pmatrix} = \begin{pmatrix} g(a_1) & g(a_2) \\ 0 & g(a_3) \end{pmatrix}$ for $g \in G$. Then

1. $A^G = Q$.
2. $B^G = \left\{ \begin{pmatrix} q_{11} & q_{12} \\ 0 & q_{33} \end{pmatrix} \middle| q_{11}, q_{12}, q_{33} \in Q \right\}$, the ring of all 2-by-2 upper triangular matrices over $Q$.
3. $\Delta = V_B(B^G) = \left\{ \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \middle| a \in A \right\} \cong A$.
4. $\Delta$ is a Galois extension of $\Delta^G$ with Galois group $G|\Delta \cong G$ and a Galois system \{1, i, j, k; 1/4, -i/4, -j/4, -k/4\}.
5. $\Delta^G = Q$ is the center of $\Delta$.
6. By (4) and (5), $B$ is a central commutator Galois extension of $B^G$.
7. The center of $B^G$ is $Q$.
8. $B^G$ is not a separable extension of its center $Q$, and so $B^G$ is not an Azumaya algebra. In fact, suppose that $B^G$ is a separable extension of $Q$. Then, there exists a separable idempotent

\[
e = \sum_{1 \leq i \leq j \leq l \leq 2} q_{ijkl} (e_{ij} \otimes e_{kl}), \tag{4.2}\]

where $e_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $e_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $e_{22} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, and $q_{ijkl} \in Q$ such that

\[
q_{ijkl} e_{ij} e_{kl} = I_2, \tag{4.3}\]

the identity 2-by-2 matrix, and $be = eb$ for all $b \in B^G$. By $e_{11} e = e e_{11}$, we have

\[
\sum_{1 \leq s \leq j \leq l \leq 2} q_{sijkl} (e_{ij} \otimes e_{kl}) = \sum_{1 \leq s \leq j \leq l \leq 2} q_{ijkl} (e_{ij} \otimes e_{11}). \tag{4.4}\]

Hence $q_{2211} = 0$ and $q_{1jk} = 0$ for all $j, k$, that is, $q_{1112} = q_{1122} = q_{1212} = q_{1222} = 0$. By $e_{12} e = e e_{12}$, we have

\[
\sum_{1 \leq s \leq j \leq l \leq 2} q_{2skl} (e_{12} \otimes e_{kl}) = \sum_{1 \leq s \leq j \leq l \leq 2} q_{ijkl} (e_{ij} \otimes e_{12}). \tag{4.5}\]

Hence $q_{22kl} = 0$ if $(k, l) \neq (1, 2)$ and $q_{ijkl} = 0$ if $(i, j) \neq (1, 2)$, that is, $q_{2211} = q_{2222} = 0$ and $q_{1111} = q_{2211} = 0$. Therefore, $e = q_{1211} (e_{12} \otimes e_{11}) + q_{2212} (e_{22} \otimes e_{12})$. Thus,

\[
I_2 = \sum_{1 \leq s \leq j \leq l \leq 2} q_{ijkl} e_{ij} e_{kl} = q_{1211} e_{12} e_{11} + q_{2212} e_{22} e_{12} = 0. \tag{4.6}\]
This contradiction shows that $B^G$ is not a separable extension of $Q$.

**Example 4.4.** Let $B = Q[i,j,k]$ be the quaternion algebra over the rational field $Q$ and $G = \{1, g_i\}$ where $g_i(x) = ix_i^{-1}$ for all $x$ in $B$. Then

(1) $B$ is a Galois extension of $B^G$ with Galois group $G$ and a Galois system $\{1, i, j, k; 1/4, -i/4, -j/4, -k/4\}$.

(2) Since $G$ is inner, $B$ is an $H$-separable extension of $B^G$.

(3) By (1) and (2), $B$ is a Galois $H$-separable extension of $B^G$.

(4) $\Delta = V_B(B^G) = Q[i]$ is not a Galois extension of $\Delta^G$ with Galois group $G|\Delta \cong G$, and so $B$ is not a central commutator Galois extension of $B^G$.

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