M-QUASI-HYPONORMAL COMPOSITION OPERATORS

PUSHPA R. SURI and N. SINGH

Department of Mathematics
Kurukshetra University
Kurukshetra - 132 119, India

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ABSTRACT. A necessary and sufficient condition is obtained for M-quasi-hyponormal composition operators. It has also been proved that the class of M-quasi-hyponormal composition operators coincides with the class of M-paranormal composition operators. Existence of M-hyponormal composition operators which are not hyponormal; and M-quasi-hyponormal composition operators which are not M-hyponormal and quasi-hyponormal are also shown.

KEY WORDS AND PHRASES. M-hyponormal, M-quasi-hyponormal, M-paranormal, normal composition operators.

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1. PRELIMINARIES.

Let \((X, S, m)\) be a sigma-finite measure space and \(T\) a measurable transformation from \(X\) into itself (that is one \(m^{-1}(E) = 0\) whenever \(m(E) = 0\) for \(E \in S\)). Then the equation \(C_T f = f o T\) for every \(f\) in \(L^2(m)\) defines a linear transformation. If \(C_T\) is bounded with range in \(L^2(m)\), then it is called composition operator. If \(X = N\) the set of all non-zero positive integers and \(m\) is counting measure on the family of all subsets of \(N\), then \(L^2(m) = l^2\) (the Hilbert space of all square summable sequences).

Let \(f_o = \frac{dm^{-1}}{dm}\) be the Radon-Nikodym derivative of the measure \(m^{-1}\) with respect to the measure \(m\),

\[
\frac{dm(ToT^{-1})^{-1}}{dmT^{-1}} = g_o, \quad \frac{dm(ToT)^{-1}}{dm} = h_o
\]

Then \(h_o = f_o \cdot g_o\).

Let \(B(H)\) denote the Banach algebra of all bounded linear operators on the Hilbert space \(H\). An operator \(T \in B(H)\) is called M-quasi-hyponormal if there exists \(M > 0\) such that

\[
M^2 T^2 - (TT)^2 \geq 0
\]
or equivalently \[ |T^2 x| \leq M |T x| \] for all \( x \) in \( H \). \( T \) is said to be M-paranormal [2] if for all unit vectors \( x \) in \( H \)
\[ |T x|^2 \leq M |T^2 x| . \]

\( T \) is said to be M-hyponormal [2] if
\[ |T x| \leq M |T x| \] for all \( x \) in \( H \).

The purpose of this paper is to generalize the results on quasi-hyponormal composition operators in [3] for M-quasi-hyponormal composition operators.

2. M-QUASI-HYPONORMAL COMPOSITION OPERATORS.

In this section we obtain a necessary and sufficient condition for M-quasi-hyponormal composition operators and then show that the class of M-quasi-hyponormal composition operators on \( L^2 \) coincides with the class of M-paranormal composition operators. We also show the existence of M-hyponormal composition operators which are not hyponormal, and M-quasi-hyponormal composition operators which are not M-hyponormal and quasi-hyponormal.

THEOREM 2.1. Let \( C_T \in B(L^2) \). Then \( C_T \) is M-quasi-hyponormal if and only if
\[ f^2 \leq M^2 h_0 . \]

PROOF. Since for any \( f \) in \( L^2 \),
\[ (C_T^2 C_T f, f) = (C_T^2 f, C_T f) = \int h_0 |f|^2 \, dm, \]
\[ = (M h_0 f, f), \]
where \( M h_0 \) is the multiplication operator induced by \( h_0 \), therefore \( C_T^2 C_T = M h_0 \). Similarly it can be seen that \( C_T^2 C_T = M f_0 \). \( C_T \) is M-quasi-hyponormal if and only if
\[ M^2 C_T^2 C_T - (C_T^2 C_T)^2 \geq 0 . \]
This implies that
\[ M^2 h_0 - M^2 f_0 \geq 0, \]
that is \( f^2 \leq h_0 \).

Hence the result.

COROLLARY. Let \( C_T \in B(L^2) \). Then \( C_T \) is M-quasi-hyponormal if and only if
\[ f_0 \leq h_0 . \]

PROOF. Since \( h_0 = f_0 \cdot g_0 \) and \( f_0 \) is positive, therefore, by above theorem we get the result.

THEOREM 2.2. Let \( C_T \in B(L^2) \). Then \( C_T \) is M-quasi-hyponormal if and only if \( C_T \) is M-paranormal.

PROOF. Necessity is true for any bounded operator \( A \). For the sufficiency, let \( C_T \) be M-paranormal, then
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\[ \left\| C_T X_{\{n\}} \right\|^2 \leq M \left\| C_T^2 X_{\{n\}} \right\| \text{ for all } n \in \mathbb{N} \]

or \[ \int |X_{\{n\}} \circ T|^2 \, dm \leq M \left( \int |X_{\{n\}} \circ T|^2 \, dm \right)^{1/2} \]

or \[ \int |X_{\{n\}}|^2 \, dm T^{-1} \leq M \left( \int |X_{\{n\}}|^2 \, dm (T^{-1}) \right)^{1/2} \]

or \[ \int f_{\circ o} \, dm \leq M \left( \int f_{\circ o} \, dm \right)^{1/2} \]

or \[ f_{\circ o}^2 (n) \leq M^2 h_{\circ o} (n) \text{ for all } n \in \mathbb{N}. \]

Hence \( f_{\circ o}^2 \leq M^2 h_{\circ o} \); \( C_T \) is M-quasi-hyponormal.

THEOREM 2.3. Let \( C_T \in B(l^2) \) and \( T: \mathbb{N} \to \mathbb{N} \) be one-to-one. Then the following are equivalent.

(i) Normal
(ii) M-hyponormal
(iii) M-quasi-hyponormal.

PROOF. (i) implies (ii), (ii) implies (iii) are always true for any bounded operator \( A \). We show that (iii) implies (i). Let \( C_T \) be M-quasi-hyponormal. Then \( \left\| C_T X_{\{n\}} \right\| \leq M \left\| C_T^2 X_{\{n\}} \right\| \) for all \( f \) in \( l^2 \). Now \( T \) is onto because if \( T \) is not onto then \( N |T(N) \) is non-empty and for \( n \in N \) \( N |T(N) \)

\[ \left\| C_T^2 X_{\{n\}} \right\| = 1 \text{ and } \left\| C_T X_{\{n\}} \right\| = 0. \]

There exists no \( M > 0 \) such that \( C_T \) is M-quasi-hyponormal which is a contradiction.

Since \( T \) is one-to-one, therefore, \( T \) is invertible, by Theorem 2.2 \[4\] \( C_T \) is invertible and \( C_T \) is normal by Theorem 2.1 \[3\].

Here we give an example of a composition operator on \( l^2 \) which is M-hyponormal but not hyponormal.

EXAMPLE 1. Let \( T: \mathbb{N} \to \mathbb{N} \) be the mapping such that

\[ T(1) = 2, \quad T(2) = 1, \quad T(3) = 2 \]

\[ T(3n+m) = n+2, \quad m = 1,2,3 \quad \text{and} \quad n \in \mathbb{N}. \]

Then \( C_T \) is not hyponormal as \( f_{\circ o} T \not\leq f_{\circ o} \) for \( n = 1 \). \( C_T \) is M-hyponormal for \( M \geq \sqrt{2} \).

EXAMPLE 2. Let \( T: \mathbb{N} \to \mathbb{N} \) be defined by \( T(1) = 2, \quad T(2) = 1, \quad T(3n+m) = n+1, \)

\[ m = 0,1,2 \quad \text{and} \quad n \in \mathbb{N}. \]

Then \( C_T \) is \( \sqrt{2} \) - quasi-hyponormal but \( C_T \) is not \( \sqrt{2} \) - hyponormal. \( C_T \) is not quasi-hyponormal also.

REFERENCES


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**Shouyang Wang,** Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China; sywang@amss.ac.cn

**K. K. Lai,** Department of Management Sciences, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong; mskklai@cityu.edu.hk